A final focus model for heavy-ion fusion driver system codes ☆

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Abstract

The need to reach high temperatures in an inertial fusion energy (IFE) target (or a target for the study of High Energy Density Physics, HEDP) requires the ability to focus ion beams down to a small spot. System models indicate that within the accelerator, the beam radius will be of the order of centimeters, whereas at the final focal spot on the target, a beam radius of the order of millimeters is required, so radial compression factors of order ten are required. The IFE target gain (and hence the overall cost of electricity) and the HEDP target temperature are sensitive functions of the final spot radius on target. Because of this sensitivity, careful attention needs to be paid to the spot radius calculation. We review our current understanding of the elements that enter into a systems model (such as emittance growth from chromatic, geometric, and non-linear space charge forces) for the final focus based on a quadrupolar magnet system. Published by Elsevier B.V.

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1. Introduction

The production of electric power from inertial fusion targets driven by heavy ion beams requires the ability to focus ion beams onto small targets. Systems models indicate that within the accelerator, the beam radius will be of the order of centimeters, whereas at the final focal spot on the target beam radii of the order of millimeters are

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required, so radial compression factors of order ten are required. Target gain, and hence the overall cost of electricity, is a sensitive function of the final spot radius on the target. In order to determine optimum beam, accelerator, target and other machine parameters, a systems code IBEAM [1] has been developed, which uses algebraic relations to model many aspects of the driver system and to develop a self-consistent model of a heavy ion-driven inertial fusion power plant. Because of the sensitivity of spot radius on target gain, careful attention needs to be paid to the part of the model that calculates the spot radius. For use as part of a previous systems assessment study [2], a model was created which estimated the contribution to the focal spot from various physical effects in the final focusing section of a driver. Since the time that model was published, numerical simulations, bench-marked by experiments, have led to a greater understanding of the processes that affect the final spot size. One of the major recent developments has been a more quantitative under-standing of neutralized ballistic transport through a final focusing chamber. Neutralized ballistic focusing experiments, (the Scaled Final Focus Experiment [3], Neutralized Transport Experiment [4]), analytic calculations, simulations using the hybrid-PIC code LSP, all point to the feasibility of focusing highly neutralized (>99%) beams. It has been found, however, that non-linear residual electric fields can cause emittance growth, and this has significant consequences for an optimized accelerator. In this paper, we update the focusing model, based on our most recent analytical and numerical calculations.

2. Elements of the model for the final spot radius

2.1. The beam edge

Our goal is to be able to estimate the spot radius over a wide range of accelerator configurations. For that reason and because the power require-ment on the target relies primarily on a knowledge of the lowest order moments, we confine ourselves to the simplest "envelope" description of the beam. The "envelope" should be regarded as the beam edge for transport within the accelerator, where the beam will likely be space charge dominated. For a uniform distribution of charge (an appropriate representation for a space charge dominated beam), the beam edges are designated a, b (in the x, y directions, respectively, transverse to the primary beam motion). For uniform distributions, the quantities a and b are twice the root mean square of the average of the respective transverse coordinates, i.e. $a = 2\langle x^2\rangle^{1/2}$ and $b = 2\langle y^2\rangle^{1/2}$. Since a is not necessarily equal to b, we allow for elliptical beams (as required by some target models), which is a generalization to the model of Ref. [2] which considered only circular beams. The envelope equations remain valid for non-uniform distributions (as long as elliptical symmetry is maintained, and the eccentricity of the elliptical density contours do not vary with radius [6], although the ellipticity may vary, with long-itudinal position z). For non-uniform distribu-tions, a and b continue to represent twice the rms averages of the particle coordinates x and y. At the target, the beam is expected to be emittance dominated, and because propagation from final focus to target tends to rotate the (x, p_x) and (y, p_y) phase space distribution (where p_x and p_y are the transverse momenta), the density distribution $n(x, y)$ on the target will ideally be Gaussian with respect to x and y. That is, $n(x, y) = n_0 \exp[-2(x^2/a^2 + y^2/b^2)]$, where n_0 is the density on-axis. For this distribution, the fraction $(1-1/e^2)$, i.e. approximately 86.4%, of the beam is contained within the ellipse with semi-axes a and b. Non-linear optics and non-linear space charge can change this value, but simulations indicate that Gaussian distributions at the target can be good approximations.

2.2. Envelope considerations

The rms envelope equations couple a and b through space charge:

$$\frac{\mathrm{d}^2 a}{\mathrm{d}z^2} = Ka + \frac{2Q}{a+b} + \frac{\varepsilon_x^2}{a^3} \tag{1}$$

$$\frac{\mathrm{d}^2 b}{\mathrm{d}z^2} = -Kb + \frac{2Q}{a+b} + \frac{\varepsilon_y^2}{b^3}. \tag{2}$$

Here z is the longitudinal coordinate in the lab frame, Q is the effective perveance, ε_x and ε_y are the unnormalized emittances in the x and y directions, and $K = B'(z)/[B\rho]$ is the magnetic quadrupole focusing function where B' is the magnetic gradient and $[B\rho]$ is the particle rigidity. This equation is valid when the beam density distribution is elliptical (and characterized by a single ellipticity at each z), and when image forces and non-linear forces can be neglected. In general, the emittances will vary with z and an auxiliary equation would be needed to integrate Eqs. (1) and (2), but for uniform density, mono-energetic beams the emittances are constant and the equations can be integrated.

Consider a region over which K is constant (e.g. in a "hard" edged quad, or within the chamber where K is zero). Multiplying Eq. (1) by a and Eq. (2) by b, adding the result and integrating allows the calculation of an energy-like constant H given by:

$$2H = a'^2 + b'^2 - Ka^2 + Kb^2$$
$$- 4Q \ln(a+b) - \frac{\varepsilon_x^2}{a^2} - \frac{\varepsilon_y^2}{b^2}. \qquad (3)$$

Here $a' = \mathrm{d}a/\mathrm{d}z$ and $b' = \mathrm{d}b/\mathrm{d}z$. After exit from the final focus magnet the beam will be converging towards the target with the following approximate conditions:

$K = 0$, $a' = -\theta_x$, $b' = -\theta_y$, $a = \theta_x d$, and $b = \theta_y d$, where d is the distance between the end of the last magnet and the focal spot, and θ_x and θ_y are the focusing half-angles. At the focal spot itself with radii $r_x = a$ and $r_y = b$, the smallest spot obtainable will be at a waist, in which case $a' = b' = 0$. (If either a' or b' are finite at the point of minimum area, we assume nevertheless that $a' \ll \theta_x$ and $b' \ll \theta_y$.) We then find

$$\theta_x^2 + \theta_y^2 \approx \frac{\varepsilon_x^2}{r_x^2} + \frac{\varepsilon_y^2}{r_y^2} + 4Q \ln\left[\frac{d(\theta_x + \theta_y)}{r_x + r_y}\right]. \qquad (4)$$

When ε_x and $\varepsilon_y = 0$, then Eqs. (1) and (2) imply $a'' - b'' = 0$, $a' - b' = \mathrm{const} = -(\theta_x - \theta_y)$, and $a - b = -(\theta_x - \theta_y)z + a_0 - b_0$. If there is a true waist at the target ($a' - b' = 0$) then $\theta_x - \theta_y = 0$,

and by symmetry

$$\theta_x^2 = \theta_y^2 \cong 2Q \ln\left[\frac{d(\theta_x + \theta_y)}{r_x + r_y}\right].$$

In the other extreme ($Q = 0$), Eqs. (1) and (2) are separable with the well-known approximate solutions

$$\theta_x^2 \approx \frac{\varepsilon_x^2}{r_x^2} \quad \text{and} \quad \theta_y^2 \approx \frac{\varepsilon_y^2}{r_y^2}.$$

We thus infer that an approximate solution for arbitrary Q, ε_x, and ε_y is given by

$$\theta_x^2 \approx \frac{\varepsilon_x^2}{r_x^2} + 2Q \ln\left[\frac{d(\theta_x + \theta_y)}{r_x + r_y}\right]$$

and

$$\theta_y^2 \approx \frac{\varepsilon_y^2}{r_y^2} + 2Q \ln\left[\frac{d(\theta_x + \theta_y)}{r_x + r_y}\right]. \qquad (5)$$

Eq. (5) is thus only approximately true for intermediate values of Q, ε_x, and ε_y. However, the sum of $\theta_x^2 + \theta_y^2$ found from Eq. (5) is consistent with the more exact relation Eq. (4), so for the purposes of this paper we adopt Eq. (5) as our generalization of the relation between the focusing angles, the final elliptical spot radii, the perveance and emittances for elliptical spots, but bearing in mind its approximate nature for intermediate values. Also, Eq. (5) is valid when Q, ε_x, and ε_y are constants. When they are not constant Eq. (5) still may be employed to approximately relate the final spot radius to the focusing angles θ_x and θ_y if the parameters are chosen appropriately. Because of the steep power of r_x and r_y in the emittance terms in Eq. (3), the final emittance should be used in Eq. (5). However, because of the logarithmic dependence of the radius in the perveance term, an appropriately averaged perveance through the fusion chamber is required.

2.3. Contributions to the final emittance

To calculate the final normalized emittance at the target, contributions to emittance dilution throughout the machine must be included. In the systems model, an estimate of the normalized emittance of each beam as it leaves the injector is

first obtained; to that are added contributions from non-linear errors in the magnetic fields of the focusing quadrupoles, chromatic aberrations and geometric aberrations in the final focus system, and the imperfect non-linear neutralization of the beam in the chamber. We expect these contributions to be uncorrelated and so the contributions are added in quadrature. Pointing errors are added separately in quadrature with the beam radius.

2.4. Injector and accelerator

The normalized emittance from the injector ε_{nxi} is given by

$$\varepsilon_{nxi} = \varepsilon_{nyi} = 2(kT/mc^2)^{1/2} r_{\text{inj}}. \tag{6}$$

Here T is the temperature of the source (about 1000 °C or ~0.1 eV for hot plate sources or ~1 eV for plasma sources), m is the ion mass, r_{inj} is the source radius (determined by using the Child–Langmuir law to calculate current, under the constraint of voltage breakdown relations). It has been found empirically [7] that imperfect beam optics increases this ideal by about a factor of two, and so in the systems code this emittance was multiplied by two to take account of this effect.

In the accelerator, non-linear magnetic fields give rise to emittance growth [8]. Also, quadrupole strength errors can create small mismatches of the beam. If the energy from those mismatches is thermalized, one may estimate the increase of emittance from each quadrupole and the result can be shown to have the same scaling as calculated in Ref. [8]. The contribution to the emittance squared from each quadrupole is

$$\Delta(\varepsilon_{nxq}^2) \approx 4a^2\beta^2 Q \left(\frac{\Delta B_q}{B_q}\right)^2. \tag{7}$$

Here ΔB_q is the component of the error magnetic field at the beam radius, which varies randomly from magnet to magnet. Contributions from all quads are summed (again in quadrature) to obtain an estimate of the final normalized emittance ε_{na} at the end of the accelerator. Nominal values of $\Delta B_q/B_q$ of 0.1% are typically assumed in system code calculations. Note that $\beta^2 Q$ is proportional to the line charge density, which, in turn, is propor-

tional to the potential energy across the beam. By adjusting the coefficient $(\Delta B_q/B_q)$, the sensitivity of spot size to emittance growth from a variety of mechanisms can be explored.

2.5. Longitudinal emittance growth

As discussed in the next section, a spread in parallel momentum will lead to an increase in emittance as the beam passes through the final focusing system. Just as in the transverse case where non-linearities give rise to transverse emittance growth, it is expected that voltage variations in the induction cell pulsers will give rise to momentum spread in the beam [9]. In this case, the normalized longitudinal emittance is proportional to the product of the momentum spread and the bunch length. At the ith induction gap, an rms momentum spread δp_i will be introduced as a result of the voltage error δV_i, such that $\delta p_i = (1/2)(p/V)\delta V_i$. (Here $p = (2mqV)^{1/2}$ is the (non-relativistic) momentum, and qV is the energy of an ion after it transits the ith gap.) It is assumed that the voltage errors ultimately are phase mixed and are not corrected soon after generation. Each gap thus contributes $\delta p_i l_i$ to the longitudinal normalized emittance where l_i is the bunch length at the ith gap. The voltage increment at each gap is $\Delta V_i = L\, dV/ds$, where L is the distance between gaps and dV/ds is the average accelerating gradient. The minimum number of pulsers at each gap is $\Delta V_i/V_{\text{pulser}}$, where V_{pulser} is the maximum voltage standoff of each pulser, e.g. around 10 kV for typical thyratrons. Assuming the errors from each pulser δV_{pulser} add stochastically, $\delta V_i = [(L\, dV/ds)V_{\text{pulser}}]^{1/2} (\delta V_{\text{pulser}}/V_{\text{pulser}})$. For purposes of obtaining estimates of momentum spread, values of $\delta V_{\text{pulser}}/V_{\text{pulser}}$ of about 1% are deemed typical of standard present technology.

The momentum spread at the end of the accelerator δp_a is thus given by

$$\delta p_a^2 = \left(\frac{l_I}{l_a}\, \delta p_I\right)^2 + \left(\frac{1}{l_a}\right)^2 \sum_i \delta p_i^2 l_i^2, \tag{8}$$

where δp_I and l_I are the momentum spread and bunch length of the beam at the end of the injector, respectively, which is calculated separately from

errors produced from the individual gaps. Incorporating the discussion preceding Eq. (8) we find

$$\frac{\delta p_a^2}{p_a^2} = \frac{1}{4V_a^2 \Delta t_a^2} \left[\delta V_I^2 \Delta t_I^2 + \sum_i \left(\frac{\delta V_{pulser}}{V_{pulser}} \right)^2 \right.$$
$$\left. \times \left(V_{pulser} L \frac{dV}{ds} \right) \Delta t_i^2 \right]. \tag{9}$$

Here Δt_I, Δt_i, and Δt_a are the pulse durations at the injector, ith gap, and accelerator end, respectively, and δV_I is the voltage error on the injector, estimated to typically be of order 10^{-3} of the injector voltage. To estimate the momentum spread δp_f at the final focus, one may assume conservation of longitudinal emittance, implying $\delta p_f / p_f = C \delta p_a / p_a$, where $C \equiv l_a/l_f$ is the compression ratio, and p_f and l_f are the ion momentum and bunch length at the final focus, respectively.

2.6. Chromatic aberrations in final focus

In Refs. [10,11] theory and simulations suggested that the contribution to the normalized emittance from chromatic aberrations in the final focus magnets alone can be written in the form

$$\varepsilon_{cx} = \alpha_{cx} d \left(\frac{\delta p}{p} \right) \theta_x^2. \tag{10}$$

Here δp is the rms longitudinal momentum deviation from the design momentum p. In Ref. [11], α_{cx} was an empirically derived constant found to be approximately 6. We have derived a set of moment equations, which treats the fractional momentum spread $\delta p/p$ as a first-order quantity in the equations of motion. The set of moment equations, in effect, generalize the envelope equations through second-order. We briefly present the derivation of the moment equations in Appendix A.

Integration of Eqs. (A.6) for particular focusing systems has found α_{cx} and α_{cy} in the range 4–12, although some systems with chromatic aberrations contributing anisotropically may have α_{cx} as small as 2.4, for example with α_{cy} approximately 6. The integration (and hence determination of α_{cx} and α_{cy}) depends sensitively on the phase of the matched envelope on which the initial value of $\langle x \delta \pi / \pi \rangle$ is assumed to be zero. In principle, this

evolves from a value of zero from the source, and is thus calculable for a given design; however, for simulations of final focus for which only the last dozen or less lattice periods are included, the indeterminacy of $\langle x \delta \pi / \pi \rangle$ leads to some uncertainty in the values of α_{cx} and α_{cy}.

2.7. Geometric aberrations in final focus

In Ref. [12], an estimate was made of third-order quantities for a heavy ion fusion focusing system. The result can be summarized in the following equation for the increase in the spot size from geometric aberrations

$$\Delta r = \frac{d\theta^3}{b\Gamma(b)^4} \approx \begin{cases} 6.9(d^2/l_{quad})\theta^3, & d > 12.5 l_{quad}, \\ 1080 l_{quad}\theta^3, & d < 12.5 l_{quad}. \end{cases} \tag{11}$$

Here d is the final focal distance, l_{quad} is the length of the final quadrupole magnet, and $b \equiv B_{quad} d / ([B\rho]\theta) \cong 1.25 d/l_{quad}$, where the latter approximate equality assumes: the thin lens approximation; the ratio of the aperture radius to the maximum radius of the beam envelope is 1.25; and the change in a' through the final lens is assumed to be equal to the convergence angle. Also, $\Gamma[b]$ is a function defined in Ref. [12] and approximated here in Eq. (17). In Ref. [12], θ represented the initial emittance divided by the design spot radius, which for small aberrations is equal to the final focus convergence angle. Eq. (11) was based on a doublet final focus system (Ref. [13]), a waist-to-waist envelope trajectory, and space charge was absent in the calculation. Particle trajectories from linear fields were used as the unperturbed orbits and non-linearities from pseudo-octupole, and B_z, arising from the fringe fields of the quadrupoles and non-paraxial terms in the equations of motion were included. Simulations have found that, for four-quadrupole systems including space charge, the spot size given by Eq. (11) overestimates the contributions of geometric aberrations by about a factor of 3, for the parameters of the Neutralized Transport Experiment [4]. We have not yet analytically reevaluated the geometric aberrations in the context of a 4 or more quadrupole final focus system in the presence

of space charge, analytically. We have parameterized our uncertainty in this quantity by introducing a factor α_g, where the emittance growth from geometric aberrations ε_{gx} is given by

$$\varepsilon_{gx} = 1080\alpha_g l_{quad}\theta_x^4, \tag{12}$$

thus giving a spot radius Δr from geometric aberrations alone of $\Delta r \sim 1080\alpha_g l_{quad}\theta_x^3$, which agrees with the scaling of Eq. (11), but where the coefficient is numerically determined for a particular magnet layout, nominally assumed to be 0.32.

2.8. Emittance growth and neutralization in the chamber

The topic of emittance growth and neutralization in the chamber has been the subject of comprehensive numerical and analytic study [15–28]. For simplicity, we have assumed ballistic neutralized transport.

2.9. Plasma plug

Simulations have shown that when a "plasma plug" (a region of ionized chamber gas in electrical contact with the walls of the chamber) is available after the beam leaves the final focus magnet, the beam space charge will draw and entrain electrons into the beam providing nearly complete neutralization of the beam space charge as it propagates to the target. However, the neutralization is not perfect and this process will only continue until the beam potential is reduced to the order $m_e v_0^2/2$ [22–24], where m_e is the electron mass. This is because in the beam frame, the electrons will have this kinetic energy relative to the beam ions, and they will not be trapped or entrained by potentials that are lower than this energy. It is straightforward to show that the resulting beam perveance Q_{c0} (in the limit of large initial beam current) is given by

$$Q_{c0} \cong \alpha_Q Z_{eff} m_e/m_i. \tag{13}$$

Here α_Q is a parameter of order unity (obtained from simulations), Z_{eff} is the effective charge state of the beam ions as they traverse the chamber (and due to stripping may not be equal to the initial

charge state), and m_i is the ion mass. At very low ion currents, the perveance of the beam itself is less than that given by Eq. (13) and simulations [21] show that for initial beam perveance less than Q_{c0}, neutralization is ineffective, and for initial beam perveance greater than Q_{c0}, the neutralized perveance approaches Q_{c0}, so that in general, the neutralized plasma plug perveance is given by

$$Q_c \approx Q_{c0}(1 - \exp(-Q_b/|Q_{c0}|)). \tag{14}$$

Here Q_b is the unneutralized beam perveance. Although the electrons provide overall charge neutralization, the distribution of electrons will not match perfectly the distribution of ions. The space charge field of the beam is therefore non-linear and the beam emittance grows as a result. Analytic work on beams with non-uniform space charge distributions [28] indicates that the emittance squared grows linearly with propagation distance according to

$$\frac{d(\varepsilon_x^2)}{dz^2} \cong \frac{\alpha_{sc}^2}{2}Q^2. \tag{15}$$

Here α_{sc} (corresponding to λ of Ref. [28]) is a parameter that depends on the distribution of charge (of order 1/2 for a variety of non-uniform charge distributions), and Q is the effective beam perveance. For propagation through the chamber, the contribution to the total emittance growth from non-linear space charge is thus given by

$$\varepsilon_{sc}^2 \approx \frac{\alpha_{sc}^2}{4}Q_c^2 d^2 + \varepsilon_0^2. \tag{16}$$

2.10. Volumetric plasma neutralization

When the target is heated by a "foot" pulse, X-rays will be emitted and can photoionize the chamber gas in the vicinity of the target. This will provide additional electrons to neutralize the beam, which can be supplied continuously along the ion beam path. Since this neutralization method does not require the electrons to become entrained with the ions, much higher space charge neutralization fractions than are given by Eqs. (13) and (14) can be obtained. In fact, the residual space charge potential may be regarded as

negligible compared with the self-focusing effect of the residual current.

In Ref. [25], a theory of the residual net current of a beam propagating through a plasma has been obtained. It is based on finding solutions of the equations of motion of an electron fluid and Maxwell's equations. It was shown that Maxwell's equations and the equations of motion of the electrons fluid have a conservation law for the generalized vorticity Ω where Ω is defined as

$$\Omega \equiv \nabla \times \mathbf{p}_e - e\mathbf{B}, \qquad (17)$$

where $\mathbf{p}_e$ is the electron fluid momentum and $\mathbf{B}$ is the self-magnetic field of the electron fluid and beam. In the limit of a long beam, where steady conditions arise, Ampere's law implies $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ so that $\nabla \times (\nabla \times \mathbf{p}_e) = \mu_0 e\mathbf{J}$. For a cylindrical symmetrical beam, Stokes theorem implies $2\pi r(\nabla \times \mathbf{p}_e)_z = \mu_0 eI(r)$, where $I(r)$ is the longitudinal current within radius r. The total net current having a radial extent the radius of the beam r_b is thus given by

$$I_{net} = \frac{2\pi}{\mu_0 e}[r(\nabla \times \mathbf{p}_e)_z]_{r=r_b} \cong \alpha_m \frac{2\pi p_e}{\mu_0 e}\left(\frac{r_b}{2r_s}\right). \quad (18)$$

The final approximate equality in Eq. (18) assumes that the scale length for the change in p_e from center to beam edge is of order r_s, and a coefficient α_m of order unity is introduced to further reflect that it is an approximation. [r_s is approximately the minimum of the "skin depth" $\delta_p = c/(e^2 n_p/\varepsilon_0 m_e)^{1/2}$ and the beam radius r_b.] At lowest order there is charge neutralization ($Z_b n_b + n_p \cong n_e$) (where n_b, n_e, and n_p are the beam, electron, and plasma ion densities respectively) and current neutralization ($qn_b v_b \cong n_e v_e$), so to lowest order $v_e \cong v_b(Z_b n_b/(n_p + Z n_b))$. This reduces to the limits $v_e \cong v_b(Z n_b/n_p)$ (if $n_b \ll n_p$) or $v_e \cong v_b$ (if $n_b \gg n_p$).

The net current is thus approximately given by

$$I_{net} \cong [Z_b n_b/(Z_b n_b + n_p)](r_b/r_s)2\pi\varepsilon_0\gamma m_e v_b c^2/e. \qquad (19)$$

The contribution to the perveance from the net current Q_m is thus

$$Q_m = \frac{-qeI_{net}}{2\pi\varepsilon_0 m_i c^3 \beta_b} = \frac{-\alpha_m}{1+f_p}\frac{qm_e}{m_i}\left(\frac{r_b}{2r_s}\right). \qquad (20)$$

Here $f_p \equiv n_p/(Z_b n_b)$. Note that the perveance is negative, resulting in a net focusing of the beam, but is less than (but can approach the order of assuming $\alpha_{sc} = \alpha_m$, $r_{b<} \sim r_s$ and $f_{p>} \sim 1$) the defocusing perveance from space charge in the case of a plasma plug (where electrons are entrained into the beam over a finite distance, in contrast to the present assumption of nearly unlimited electrons allowed to be dragged transversely into the beam over the entire length of propagation of the beam). (See Ref. [25] for a complete and more rigorous derivation of Eq. (20).) This model may be appropriate for the steady-state portion of the main pulse, for which a "foot" pulse has heated the target and preionized the plasma in the vicinity of the target, and a plasma "plug" has provided a plasma near the entrance of the chamber. Thus, Eq. (20) may be used to estimate the final perveance in Eq. (5), for the main pulse of the target. Note that the scaling for volumetric plasmas has been found to be quite different from plasma plugs [27]. In Ref. [27] it is shown that the effective perveance is quite independent of initial beam perveance (in contrast to Eq. (14)) and that the neutralization is most dependent on whether the quantity $\omega_p \Delta t$ is $\ll$ or $\gg 1$, the latter condition, required for validity of Eq. (20). Here ω_p is the electron plasma frequency and Δt is the ion beam pulse duration.

For determining emittance growth in the chamber, it is still appropriate to use Eqs. (14) and (16), since the "plasma plug" perveance will determine the emittance growth for a large fraction of the chamber.

In applying these equations, one should bear in mind that these are approximate relations, intended to obtain the correct scaling over variations in accelerator or beam parameters. At this point of our understanding, detailed simulations are still needed to accurately predict the spot radius for a given set of beam and plasma parameters.

3. Summary of model

For an approximate determination of the semi major axes of the final elliptical spot in a systems code (such as IBEAM), a knowledge of the beam

emittance and fractional momentum spread on entrance to the final focus section, perveance while propagating through the chamber, final convergence angle θ, and final focal distance d, are needed. The final emittance may be calculated by adding in quadrature estimates of the emittance from the various sources

$$\varepsilon_x^2 = \varepsilon_{xa}^2 + \alpha_{cx}^2 d^2 \left(\frac{\delta p}{p}\right)^2 \theta_x^4$$
$$+ 1.16 \times 10^6 \alpha_g^2 l_{quad}^2 \theta_x^8 + \alpha_{sc}^2 Q_c^2 d^2$$

and

$$\varepsilon_y^2 = \varepsilon_{ya}^2 + \alpha_{cy}^2 d^2 \left(\frac{\delta p}{p}\right)^2 \theta_y^4$$
$$+ 1.16 \times 10^6 \alpha_g^2 l_{quad}^2 \theta_y^8 + \alpha_{sc}^2 Q_c^2 d^2. \qquad (21)$$

The approximate solutions to the envelope equations (Eq. (5)) can then be used to estimate the elliptical spot radii.

3.1. Example: 4.4 GeV, 1.1 kA bismuth beam

To demonstrate the magnitude of the various contributions to the focal spot we take as a "typical" example a beam from a driver with ion species bismuth (209 amu), a single beam current of 1.9 kA, and ion energy 4.0 GeV. These parameters are similar to those for a main pulse of the "Robust Point Design" of Ref. [30]. The 1.9 kA is a result of a factor of 17 increase in current in the drift compression section. For a source with radius 5 cm and temperature 1.0 eV the normalized emittance of such a beam would be 0.23 mm mrad, which, allowing for optics imperfections, may be ~0.46 mm mrad at the end of a 15 cm, 600 kV injector diode. The current of such a beam would be 0.6 A, through the end of a 2 MV ($\beta = 0.00453$) injector.

Accelerating, in a magnetic quadrupole lattice from 2 MeV to 4 GeV, the line charge density increases by about a factor of 2.5, so the quantity $\sim 4\beta^2 Q a^2 = 2q\lambda a^2/(\pi\varepsilon_0 mc^2)$ increases by this amount also. For a 2 cm beam, at 0.44 μC/m, and 0.1% $\Delta B_q/B_q$ the normalized emittance growth per quadrupole would be 8×10^{-3} mm mrad. Taking into consideration $d \ln \lambda / d \ln V = 0.12$,

with an accelerator with ~675 quads, the normalized emittance has increased by only 0.33 mm mrad, which gives a combined emittance from injector and quads to be 0.57 mm mrad. This is somewhat ideal, but sets goals for what is possible. Similarly, the fractional parallel momentum spread can be estimated from Eq. (9). For an initial pulse of 30 μs that is compressed to 170 ns in the accelerator and then down to 10 ns at the final focus, using $d \ln \Delta t / d \ln V = -0.62$, we find a contribution to $\delta p / p_f$ of 6×10^{-4}, from errors in the induction gaps. For an injector voltage error of 10^{-3}, we find a contribution to $\delta p / p_f$ of 4.5×10^{-4}, so that the predicted total $\delta p / p_f$ is 7.5×10^{-4}.

With this fractional momentum spread, a final focus distance of 6 m, a final focusing angle θ of 10 mrad, and α_{cx} of 4, then ε_{ncx} is 0.35 mm mrad. With these parameters, and $\alpha_g = 0.32$, a 1.6 m final magnet, then Eq. (18) (emittance from geometric aberrations) suggests that $\varepsilon_{ng} = 2.0$ mm mrad. (Note that in calculating chromatic and particularly geometric aberrations, a magnet layout is generally needed. For Ref. [30] the layout involved a number of constraints based on spot size, magnet construction difficulty, magnet replacement rate (minimization of magnet activation), and thresholds for angular array extent (dictated by target considerations) and maximum tolerable beam ellipticity (to avoid additional emittance growth).) Finally, while propagating in the chamber, for this example, from Eqs. (19) and (20), a final perveance of 5.2×10^{-6}, could be reached, if $\alpha_Q Z_{eff} = 2$, and a "plasma plug" is the source of neutralization. This would lead to a normalized emittance contribution $\varepsilon_{nsc} = 3.1$ mm mrad (if $\alpha_{sc} = 0.5$). The sum of all normalized emittances is 3.7 mm mrad, in this example, which, when included with the perveance term in Eq. (5), yields a final spot size of r_x of 2.2 mm. If there is photoionization of chamber gas by the heated target, then the plasma density may be as high as 3.5×10^{13} cm^{-3}, and the beam density may be as high as 7.1×10^{12} cm^{-3}, so that f_p may be of order unity and $r_b/2r_s$ may also be of order unity, so that the final perveance may be $\sim -5 \times 10^{-6}$. If the emittance growth is assumed to be the same (due to the same conditions as occur in the plasma plug

throughout most of the chamber, but that the final perveance determines the contribution to the space charge term in Eq. (5)) then the final spot radius would be 1.6 mm. Both results would meet the target requirements. This example illustrates the most recent realization that neutralization in the chamber (and the associated non-linear electric fields) may be the dominant source of beam emittance throughout the entire system. When the equilibrium perveance is larger than the bare beam perveance, then the final perveance and emittance growth (and hence spot size) is largely independent of beam parameters. However, for small enough beams neutralization (and associated emittance growth) may not be significant and the other terms in Eq. (27) may be the largest contributors.

4. Improvements to the model

Corrections: Corrections of beam aberrations using non-linear optics have been suggested by several authors and have not been included in the focal spot model. These include corrections to geometric aberrations using octupoles [32] and corrections to chromatic aberrations (using combinations of sextupoles/dipoles) [33]. Time-dependent focusing has also been suggested to correct chromatic aberrations [31,34,35]. This latter idea is that at the beginning of the drift compression, there is a strong correlation between longitudinal position and velocity. One could then, in principle, give each velocity class upstream (before drift compression) a time-dependent variation in the quadrupole field strength such that when that velocity class passes through the final focus optics, each velocity class will focus to the correct spot, thus minimizing chromatic aberrations [31]. Two possible problems are (1) the correct quadrupole variations upstream to optimize the focal spot may cause the beam to be mismatched, and (2) space charge may cause various velocity classes to be mixed at the final focus so the correction may not be so effective, (but there could still be some benefit). Multiple corrections might also be made during the drift compression. Correcting closer to final focus reduces the precision needed, as there

are fewer betatron periods between the correction point and the final focus, but the correlation between velocity and longitudinal position is going down, so the corrections may become less effective the closer the beam is to final focus. Also, as the pulse duration decreases towards 10 ns, the technological challenges may increase. The efficacy of these various correction schemes has yet to be determined. Assuming that they are successful, their impact on the overall configuration of an optimized accelerator and on the cost of electricity can be determined by setting α_{cx}, α_{cy}, α_g to zero or to some small value.

4.1. Emittance growth model

In addition to voltage errors contributing to the longitudinal emittance, simulations show that a transverse/longitudinal instability will grow until the longitudinal temperature is roughly a third to half of the transverse temperature. For an initial transverse temperature kT of 2 eV, an injection voltage V_I of 2 MV, the corresponding $\delta p/p \sim (kT/[4qV_I])^{1/2} \sim 5 \times 10^{-4}$, so it is apparent that injector voltage waveform errors are initially somewhat more important than thermalization of transverse temperature, but this should be tracked throughout the accelerator. Another source of longitudinal emittance growth is timing jitter from the confining ("ear") waveform pulsers. This will be a source of space charge waves that can propagate to the interior of the beam. Even if the waves do not thermalize they will contribute to chromatic aberrations to the focal spot if not corrected. A simple model to account for jitter could be incorporated into the emittance growth model.

5. Conclusion

We have constructed a source-to-target model of the emittance evolution of a heavy ion beam. We have used the envelope equations to estimate the semi-major axes of the beam spot radius on the target including the combined effects of emittance contributions from the injector temperature, quadrupole focusing errors in the accelerator,

chromatic and geometric effects in final focus, non-linear and incomplete neutralization in the chamber. The chromatic effects are determined from an estimate of longitudinal velocity spread arising from injector and induction gap voltage errors through the accelerator. We have estimated the size of the various effects for one example set of parameters, and have found that the contribution from non-linear space charge effects in the chamber can be the dominant source of emittance growth, and is insensitive to beam parameters. Further refinement of these estimates, using PIC codes for benchmarking, is in progress.

Appendix A. Moment equations for chromatic aberrations

We start with the equation of motion for a beam ion in a quadrupolar external field and include the effects of space charge distributed in an elliptical distribution. The x-component of the equation of motion is

$$\frac{\mathrm{d}p_x}{\mathrm{d}t} = q(E_x + v_z B_y - v_y B_z). \tag{A.1}$$

Here E_x is the x component of the space charge field of the beam, and B_y and B_z are the y and z components of the magnetic field of the quadrupole. Expanding through second order in x', y', $k_{\beta 0}x$, $k_{\beta 0}y$, $\delta p/p$ yields

$$x'' + \left(\frac{1}{\gamma v_{z0}} \frac{\mathrm{d}}{\mathrm{d}z}(\gamma v_z) \right) x'$$
$$\cong \frac{qB'}{\gamma m v_{z0}} x \left(1 - \frac{\delta p}{p} \right)$$
$$+ \frac{q\lambda}{4\pi\varepsilon_0 m v_{z0}^2} \frac{(x - \bar{x})(1 - 2\delta p/p)}{(\Delta x^2 + [\Delta x^2 \Delta y^2]^{1/2})}. \tag{A.2}$$

This equation (when $\mathrm{d}(\gamma v_z)/\mathrm{d}z = 0$) can be written as

$$x'' \cong K_{xx} x + K_{xx1} x \delta p/p. \tag{A.3}$$

Similarly, the y-equation of motion can be written as

$$y'' \cong K_{yy} y + K_{yy1} y \delta p/p. \tag{A.4}$$

Here

$$K_{xx} = \pm \frac{B'}{[B\rho]_0} + \frac{2Q}{2(\Delta x^2 + [\Delta x^2 \Delta y^2]^{1/2})}$$
$$K_{xx1} = -\left[\pm \frac{B'}{[B\rho]_0} + \frac{2Q}{2(\Delta x^2 + [\Delta x^2 \Delta y^2]^{1/2})} \right]$$
$$K_{yy} = \frac{\mp B'}{[B\rho]_0} + \frac{Q}{2(\Delta y^2 + [\Delta x^2 \Delta y^2]^{1/2})}$$
$$K_{yy1} = -\left[\frac{\mp B'}{[B\rho]_0} + \frac{2Q}{2(\Delta y^2 + [\Delta x^2 \Delta y^2]^{1/2})} \right]. \tag{A.5}$$

Here, B' = magnetic gradient, $[B\rho]$ = ion rigidity $= p/q$ and Q = perveance $= q\lambda/(2\pi\varepsilon_0 \gamma_0^3 m v_{z0}^2)$.

Let $\delta \equiv \delta p/p$. Noticing that

$$\frac{1}{1+\delta} = 1 - \delta + \delta^2 + \dots$$

and

$$\frac{1}{1-\delta} = 1 + \delta + \delta^2 + \dots$$

so that

$$\frac{1}{1-\delta} - \frac{1}{1+\delta} = 2\delta + 2\delta^3 + \dots$$

and also

$$\frac{\delta}{1+\delta} = 1 - \frac{1}{1+\delta}.$$

We may perform averages over the distribution function and obtain the set of moment equations

$$\frac{\mathrm{d}}{\mathrm{d}s}\langle x^2 \rangle = 2\langle xx' \rangle$$

$$\frac{\mathrm{d}}{\mathrm{d}s}\langle xx' \rangle = \langle x'^2 \rangle + K_{xx}\langle x^2 \rangle + \frac{K_{xx1}}{2}\left[\left\langle \frac{x^2}{1-\delta} \right\rangle - \left\langle \frac{x^2}{1+\delta} \right\rangle \right] + O(x^2\delta^3)$$

$$\frac{\mathrm{d}}{\mathrm{d}s}\langle x'^2 \rangle = 2K_{xx}\langle xx' \rangle + K_{xx1}\left[\left\langle \frac{xx'}{1-\delta} \right\rangle - \left\langle \frac{xx'}{1+\delta} \right\rangle \right] + O(xx'\delta^3)$$

Table 1
Summary of constants in emittance growth model

Quantity	Nominal value	Comments
α_{cx}, α_{cy}	4	Normalized emittance growth from chromatic aberrations; dependent on final focus magnet layout, and initial correlation of x and $\delta p/p$
α_g	0.32	Normalized emittance growth from geometric aberrations; dependent on final focus magnet layout
α_Q	2	Normalization constant in final perveance for "plasma plug" neutralization methods
α_m	1	Normalization in final perveance for "zero vorticity" model
α_{sc}	0.5	Normalization in emittance growth rate due to space charge non-linearity; depends on distribution of space charge

$$\frac{d}{ds}\left\langle \frac{xx'}{1+\delta} \right\rangle = \left\langle \frac{x'^2}{1+\delta} \right\rangle + K_{xx}\left\langle \frac{x^2}{1+\delta} \right\rangle$$
$$- K_{xx1}\left\langle \frac{x^2}{1+\delta} \right\rangle + K_{xx1}\langle x^2 \rangle$$

$$\frac{d}{ds}\left\langle \frac{x'^2}{1+\delta} \right\rangle = 2K_{xx}\left\langle \frac{xx'}{1+\delta} \right\rangle + 2K_{xx1}\langle xx' \rangle$$
$$- 2K_{xx1}\left\langle \frac{xx'}{1+\delta} \right\rangle. \tag{A.6}$$

The equations for y are identical with the substitution $y \to x$, in all variables and subscripts. Note that the error terms in Eq. (A.6) are two orders of δ higher than the effective order of the term in brackets and are thus dropped. In practice, this limits the integration distance over which the equations may be used, but over a short number of lattice periods as in a final focus system, the system of equations is quite accurate. When candidate final focus systems are evaluated, we find that the scaling $\varepsilon \propto \Delta p/p$ is satisfied, and when the factors θ_x, θ_y, and d are included in the normalization of the relation, the factors α_{cx} and α_{cy} depend on the

particular final focus system, (and on the initial phase of the matched period for which the value of $\langle x\delta \rangle = 0$ is assumed), but are in the range 4–12. Table 1 summarizes the dimensionless parameters of order unity used in calculating the final ion beam emittance on the spot. Generally, simulations are required to determine each α.

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